

# Assignment 15

## DEFINITION OF GALOIS GROUP

1. Let  $E/k$  be a field extension. Show that every  $\sigma \in \text{Gal}(E/k)$  is an invertible  $k$ -linear map of the  $k$ -vector space  $E$ .
2. Show that  $\text{Gal}(\mathbb{C}/\mathbb{R}) = \{\text{id}, \sigma\}$ , where  $\sigma$  is the complex conjugation.
3. Determine all irreducible polynomials of degree 1, 2, 3, 4, 5 in  $\mathbb{F}_2[X]$ .
4. Show that  $X^4 + 1 \in \mathbb{Q}[X]$  is irreducible. Show that  $X^4 + 1$  is reducible in  $\mathbb{F}_p[X]$  for every prime  $p$ .