# Assignment 19

## Normality and separability

1. Let $K/k$ be a field extension and $f \in k[X]$. Prove that $f$ is separable as a polynomial in $k[X]$, then it is separable as a polynomial in $K[X]$. Does the converse hold?

2. Let $f \in k[X]$ be a monic polynomial which splits and suppose that $\sigma \in \mathrm{Aut}(k)$ fixes each root of $f$. Prove that $\sigma$ fixes all the coefficients of $f$.

3. Let $E/k$ be a splitting field of $f \in k[X]$ and consider an extension $k'$ of $k$ and the splitting field $E'$ of $f$ over $k'$. Show that each $\sigma \in \mathrm{Gal}(E'/k')$ satisfies $\sigma(E) = E$ and that the resulting homomorphism
$$\begin{aligned} \mathrm{Gal}(E'/k') &\longrightarrow \mathrm{Gal}(E/k) \\ \sigma &\longmapsto \sigma|_E \end{aligned}$$
is injective.

4. Let $E/k$ be a finite field extension. Show that $E/k$ is normal if and only if every irreducible polynomial $f \in k[X]$ which has a root in $E$ splits completely over $E$.

5. Show that $\mathrm{Aut}(\mathbb{R}) = \{\mathrm{id}_{\mathbb{R}}\}$.