$$f(t) = e^{\alpha t} \sin\beta t, \quad 0 < \alpha < Re(s)$$

$$\mathcal{L}(f)(s) := \int_0^\infty e^{-st} e^{\alpha t} \sin\beta t \, dt$$

$$= \int_0^\infty e^{-t(s-\alpha)} \sin\beta t \, dt$$

$$= \left[\overset{u}{\underset{u}{e^{-t(s-\alpha)}}} \cdot \overset{v'}{\underset{v}{\frac{\cos\beta t}{-\beta}}} \right]_0^\infty - \int_0^\infty \underset{u' \cdot v}{\cancel{-}(s-\alpha) \cdot e^{-t(s-\alpha)} \frac{\cos\beta t}{\cancel{-}\beta}} \, dt$$

$$= 0 - \frac{e^0 \cdot \cos 0}{-\beta} - \int_0^\infty \underset{u}{(s-\alpha) e^{-t(s-\alpha)}} \underset{v'}{\frac{\cos\beta t}{\beta}} \, dt$$

$$= \frac{1}{\beta} - \left[\underset{u}{(s-\alpha) e^{-t(s-\alpha)}} \cdot \underset{v}{\frac{\sin\beta t}{\beta^2}} \right]_0^\infty$$

$$+ \int_0^\infty \underset{u'}{-(s-\alpha)^2 e^{-t(s-\alpha)}} \underset{v}{\frac{\sin\beta t}{\beta^2}} \, dt$$

$$= \frac{1}{\beta} - \frac{(s-\alpha)^2}{\beta^2} \int_0^\infty e^{-t(s-\alpha)} \sin\beta t \, dt$$

$$\Rightarrow \left[1 + \frac{(s-\alpha)^2}{\beta^2} \right] \int_0^\infty e^{-t(s-\alpha)} \sin\beta t \, dt = \frac{1}{\beta}$$

$$\Rightarrow \int_0^\infty e^{-t(s-\alpha)} \sin\beta t \, dt = \underline{\underline{\frac{\beta}{(s-\alpha)^2 + \beta^2}}}$$