

# Probability and Statistics

## Exercise sheet 11

**Exercise 11.1** Suppose that  $X_1, \dots, X_n$  form a random sample from a Poisson distribution for which the mean is unknown. Determine the maximum-likelihood estimator of the standard deviation of the distribution.

**Exercise 11.2** Suppose that  $X_1, \dots, X_n$  form a random sample from a distribution for which the probability density function  $f(x|\theta)$  is as follows:

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1} & \text{for } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Also, suppose that the value of  $\theta$  is unknown ( $\theta > 0$ ). Find the M.L.E. of  $\theta$ .

**Exercise 11.3** In a lake we want to estimate the amount of a certain type of fish. For this we mark 5 fishes and we let them mix with the others, when they are well mixed we fish 11, and we realize that there are 3 marked and 8 non-marked. What is the maximum-likelihood estimator for the amount of fishes?

**Exercise 11.4** A gas station estimates that it takes at least  $\alpha$  minutes for a change of oil. The actual time varies from customer to customer. However, one can assume that this time will be well represented by an exponential random variable. The random variable  $X$ , therefore, possesses the following density function

$$f(t) = e^{-t} \mathbb{1}\{t \geq \alpha\},$$

i.e.  $X = \alpha + Z$  where  $Z \sim Exp(1)$ . The following values were recorded from 10 clients randomly selected (the time is in minutes):

4.2, 3.1, 3.6, 4.5, 5.1, 7.6, 4.4, 3.5, 3.8, 4.3.

Estimate the parameter  $\alpha$  using the maximum likelihood estimator.

**Exercise 11.5** Suppose that  $X_1, \dots, X_n$  form a random sample from a normal distribution for which both the mean and the variance are unknown. Find the M.L.E. of the 0.95 quantile of the distribution, that is, of the point  $\theta$  such that  $\mathbb{P}[X < \theta] = 0.95$ .

The 0.95 quantile of a standard normal distribution is  $1.645 =: \theta_0$ .