D-MATH | Algebraic Topology II | FS 17

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Exercise Sheet 10

1. Let X be a CW complex and A a compact subset of X. Show that A is contained in a finite subcomplex of X.

2. Let X be a CW complex with filtration $X_0 \hookrightarrow X_1 \hookrightarrow X_2 \hookrightarrow \dots$. For each integer $k \geq 0$, prove that the induced map $\pi_k(X_n) \longrightarrow \pi_k(X_{n+1})$ is surjective for $n = k$ and an isomorphism for $n > k$.

3. Show that a CW complex is contractible if it is the union of an increasing sequence of subcomplexes $X_1 \subseteq X_2 \subseteq \dots$ such that each inclusion $X_i \hookrightarrow X_{i+1}$ is nullhomotopic.

4. **a)** Explain how to define the infinite sphere S^∞ as a CW complex.

b) Prove that S^∞ is contractible.

5. Show that an n-connected, n-dimensional CW complex is contractible.

6. Let X and Y be homotopy equivalent CW complexes, both without $(n+1)$-cells. Prove that the n-skeletons of X and Y are also homotopy equivalent.

7. Consider the Hopf map $h : S^3 \longrightarrow S^2$ and consider $X := D^4 \sqcup S^2 / \sim$ where $\sim$ identifies the points of $S^3 \subseteq D^4$ with their image via h in S^2. Compute $\pi_k(X)$ for $k = 0, 1, 2, 3$. Can you say something about $\pi_4(X)$?