

Exercise Sheet 12

1. Let $X_n = S^1 \vee S^n$.
 - a) Describe the universal covering of X_n and the monodromy action of $\pi_1(X_n)$ on it.
 - b) Deduce that there is a group isomorphism $\pi_n(X_n) \cong \mathbb{Z}[t, t^{-1}]$ for $n \geq 2$ and describe the natural action of $\pi_1(X_n)$ on $\mathbb{Z}[t, t^{-1}]$.

2. Let X and Y be CW complexes and consider the suspension sequence

$$[X, Y] \xrightarrow{\Sigma} [\Sigma X, \Sigma Y] \xrightarrow{\Sigma} [\Sigma^2 X, \Sigma^2 Y] \longrightarrow \dots$$

Suppose that X is finite. Use Freudenthal suspension theorem to show that these maps eventually become isomorphisms.

3. Let E be a contravariant functor from the category of basepointed CW complexes to the category of abelian groups, satisfying the homotopy and additivity axioms. Let X and K be CW complexes and $f, g : \Sigma X \rightarrow K$ two basepoint-preserving maps. Prove that $E(f + g) = E(f) + E(g)$.