D-MATH | Algebraic Topology II | FS 17

Prof. Peter S. Jossen

Exercise Sheet 13

1. Show in detail how adjunction gives the isomorphism

$$\pi_n(F(x_0, X, B)) \cong \pi_{n+1}(X, B, x_0)$$

mentioned in class, where X is a CW complex, $x_0 \in B \subseteq X$ and $F(x_0, X, B)$ is the set of paths in X with startpoint x_0 and endpoint in B.

2. Let X and Y be pointed CW complexes.

a) Show that the map $\pi_i(X \vee Y) \longrightarrow \pi_i(X \times Y)$ induced by the inclusion is always surjective.

b) Show that we obtain short exact sequences

$$0 \longrightarrow \pi_{i+1}(X \times Y, X \vee Y) \longrightarrow \pi_i(X \vee Y) \longrightarrow \pi_i(X \times Y) \longrightarrow 0.$$

c) Does the above short exact sequence split for $i = 1$ and $X = Y = S^1$?

3. Let $q \geq 1$. Let X be a connected CW complex and Y a q-connected CW complex.

a) Prove that the map $\pi_i(X) \longrightarrow \pi_i(X \vee Y)$ is an isomorphism for $i < q$.

b) Show that $\pi_2(X \times Y, X \vee Y) = 0$.

4. Show that the map $\pi_3(D^2, S^1) \longrightarrow \pi_3(D^2/S^1)$ is not surjective.

5. Show that $\pi_1(S^2 \vee S^1, S^1) = 0$, and that the map $\pi_2(S^2 \vee S^1, S^1) \longrightarrow \pi_2(S^2 \vee S^1/S^1) \cong \pi_2(S^2)$ is surjective but not injective.

6. Determine $\pi_{2n-1}(S^n \vee S^n)$ for $n \geq 2$.