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Exercise Sheet 3

1. Let F be a free group on two generators a and b. How many subgroups of F have index 2? Specify generators for each of these subgroups.

2. Prove that a non-trivial normal subgroup N with infinite index in a free group F cannot be finitely generated.

3. Prove the following statements about topological spaces:

a) Any subspace of a weak Hausdorff space is weak Hausdorff.

b) Any closed subspace of a k-space is a k-space.

c) An open subset U of a compactly generated space X is compactly generated if each point has an open neighborhood in X whose closure contained in U.

4. (*) In this exercise, we want to find the fundamental grupoid of S^1 by using the groupoid version of van Kampen's theorem.

a) Let $\mathcal{C}$ be a category. Define a diagram in $\mathcal{C}$ as a functor from a suitable category to $\mathcal{C}$. What is a morphism of diagrams? What is an isomorphism of diagrams? Prove: given two isomorphic diagrams, their colimits (if they exist) are isomorphic.

b) Explain how the colimit of a finite diagram of grupoids can be constructed.

c) Choose a finite open covering of S^1 made of a small number of simply connected opens U_i and stable under intersections.

d) Compute $\Pi(S^1)$ using van Kampen's theorem. [*Hint:* Replace the $\Pi(U_i)$ with appropriate finite groupoids, use **a)**]