Exercise Sheet 4

1. Show that a cofibration $i : A \longrightarrow X$ is an embedding with closed image (e.g., the inclusion of an closed subset). Give an example of a closed embedding which is not a cofibration.

2. Let $i : A \longrightarrow X$ be a cofibration, where A is a contractible space. Prove that the quotient map $X \longrightarrow X/A$ is a homotopy equivalence.

3. Let $i : A \longrightarrow X$ be a cofibration and Y a topological space. Is $i \times \mathrm{id}_Y : A \times Y \longrightarrow X \times Y$ a cofibration as well?

4. Let $\{U_i\}_{i \in I}$ be an open covering of a topological space X which is stable under finite intersections, and $A \subseteq X$ a closed subspace. Suppose that $A \cap U_i \hookrightarrow U_i$ is a cofibration for each $i \in I$. Is the inclusion $A \hookrightarrow X$ a cofibration as well?

5. For each of the following inclusion maps, decide whether they are cofibrations or not:

- $\{x \in \mathbb{R}^n : \|x\| < 1\} \hookrightarrow \mathbb{R}^n$;
- $\{x \in \mathbb{R}^n : \|x\| \leq 1\} \hookrightarrow \mathbb{R}^n$;
- $S^1 \hookrightarrow \mathbb{C}$;
- $S^1 \times \{1\} \hookrightarrow S^1 \times S^1$.

6. Let $A \hookrightarrow B$ and $B \hookrightarrow X$ be cofibrations. Show that the composite $A \hookrightarrow X$ is a cofibration.