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Exercise Sheet 5

1. Read the proof of the Theorem in Chapter 6, Section 4 of Peter May's book *A Coincise Course in Algebraic Topology* and rewrite it in detail.

2. Show that the composition of two fibrations is a fibration.

3. **a)** Show that the projection map

$$\begin{aligned} \mathrm{GL}_2(\mathbb{R}) &\longrightarrow \mathbb{R}^2 \setminus \{0\} \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix} &\mapsto (a, b) \end{aligned}$$

is a fibration.

b) Is it locally trivial? Is it trivial?

c) Compute the fibre translation action of $\pi_1(\mathbb{R}^2 \setminus \{(0,0)\}, \{(1,0)\})$ on $p^{-1}(1,0) = \left\{ \begin{pmatrix} 1 & 0 \\ c & d \end{pmatrix} : d \neq 0 \right\}$.

4. Let X_n be the topological space whose elements are ordered tuples $(L_1, \ldots, L_n)$ of n lines in general position $L_1, \ldots, L_n \subseteq \mathbb{R}^2$.

a) Give a precise definition of X_n.

b) Prove that the map $X_n \longrightarrow X_{n-1}$ deleting the last line is a fibration.

c) Try to compute the action of $\pi_1(X_1, L_1 : x_1 = 0)$ on $p^{-1}(L_1)$.

The spaces X_n are examples of configuration spaces or, in other terms, of moduli spaces.