1 The adjunction is given by

$$\left[(I^n, \partial I^n), (F(x_0, X, B), c_{x_0})\right] \longrightarrow \left[(I^{n+1}, \partial I^{n+1}, J^{n+1}), (X, B, x_0)\right]$$
$$\left(x \longmapsto \varphi_x(t)\right) \longmapsto \left[(x,t) \longmapsto \varphi_x(t)\right]$$

First of all this is well defined as $\varphi_x(t) \in X$ for all x, t and on $\partial I^n \times I$ $\varphi_x(t) = x_0$ and $I^n \times \partial I$ $\varphi_x(t) \in B$. Since $\left[(x,t) \longmapsto \varphi(x,t)\right] \longrightarrow \left[x \longmapsto \varphi(x, -)\right]$ is an inverse, this map is bijective.

For $n \geq 1$ it furthermore respect the group laws, as we "stack" two maps in the "t"-coordinate.

Sheet 13

3a We look at the long exact sequence given by $(X\vee Y, X)$

$$\pi_{i+1}(X\vee Y, X) \to \pi_i(X) \to \pi_i(X\vee Y) \to \pi_i(X\vee Y, X)$$

It suffices to show that $\pi_i(X\vee Y, X)=0$ for $i\le q$. We do this using excision:
$(Y,\{pt\})$ is q-connected, $(X,\{pt\})$ is connected and thus we have $\pi_i(Y) \xrightarrow{\sim} \pi_i(X\vee Y, X)$ for $i\le q$.
Since Y is q-connected, we are done for $i<q$. For $i=q$, we just need to show that $\pi_q(X)\to\pi_q(X\vee Y)$ is injective. Assume that the image of a representative of a class of $\pi_q(X)$ is homotopic to the constant map, then the same homotopy would do the job in X. Hence $\pi_i(X)\to\pi_i(X\vee Y)$ is an iso for $i\le q$.

b) We look at the short exact sequence which we know from 2)

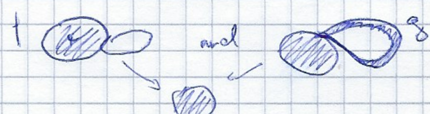
$$0 \to \pi_2(X\times Y, X\vee Y) \to \pi_1(X\vee Y) \to \pi_1(X\times Y) \to 0$$

$$\pi_1(X\vee Y)\cong\pi_1(X) \xrightarrow{\sim} \pi_1(X)\times\pi_1(Y) = \pi_1(X), \qquad \pi_1(X\times Y)=\pi_1(X)\times\pi_1(Y)$$

Thus $\pi_2(X\times Y, X\vee Y)=0$.

5. S^1 is connected, S^2 is 1-connected, hence by excision $\pi_2(S^2\vee S^1, S^1) = \pi_2(S^2)=0$
$\pi_2(S^2\vee S^1, S^1)\to\pi_2(S^2)$ is surjective, since for any $[f]\in\pi_2(S^2)$ the image under $\pi_2(S^2)\hookrightarrow\pi_2^{*}(S^2\vee S^1, S^1)$ gets mapped to $[f]$.

For injectivity we look at the maps



The map just sends any objects (or rather representative) to its image in S^2. So the images of $[f]$ and $[g]$ are the same. However $[f]$ and $[g]$ are not the same, as the loop in g cannot be [illegible] as it is fixed onto the base point.

4. We first show $\pi_3(D^2, S^1)=0$. For that we look at the exact sequence of (D^2, S^1)

$$0=\pi_3(D^2)\to\underset{\Downarrow\\ =0}{\pi_3(D^2,S^1)}\to\underset{\|\\ 0}{\pi_2(S^1)}\to\pi_2(D^2)=0$$

Furthermore we have $\pi_3(D^2/S^1) = \pi_3(S^2)$ which is $=\mathbb{Z}$ as a consequence of the Hopf fibration. Thus the map $\pi_3(D^2,S^1)\to\pi_3(D^2/S^1)$ can not be surjective.