1. Let $K$ be a complete valued field with non-discrete topology and let $E$ be a Hausdorff topological vector space over $K$ of finite dimension $n$. Prove that $E \cong K^n$ as topological vector spaces. [Hint: Induction on $n$]

2. Let $A$ be a DVR with residue field $k$ and let $f \in A[X]$ be an irreducible polynomial. Let $\bar{f} \in k[X]$ be the reduction of $f$ modulo $m_A$ and suppose that the extension $k' := k[X]/(\bar{f})$ is a separable field extension of $k$. Prove that the ring $A[X]/(f)$ is a DVR with residue field $k'$.

3. a) Give a statement of the quadratic reciprocity law.
   
   b) Find for which primes $p$ the polynomial $f(X) = X^2 - 11$ has a root in $\mathbb{Q}_p$.
   
   c) Let now $p = 5$. For each root $\alpha$ of $f$, compute the coefficients $a_i \in \{0, 1, \ldots, 4\}$ of the expansion $\alpha = \sum_{i \in \mathbb{Z}} a_i 5^i$ for $i < 5$.

4. Let $K$ be a complete discrete value field and $L/K$ a finite algebraic extension. Let $T \subseteq L$ be the maximal unramified extension of $K$ in $L$. Let $k \subseteq t \subseteq \ell$ be the associated residue field extensions. Prove: $t$ is the maximal separable extension of $k$ in $\ell$. 