TABLE 5.3

Field	totally decomposed prime numbers p	ramified prime numbers p
$\mathbb{Q}(\sqrt{-1})$	$p \equiv 1 \bmod 4$	$p = 2$
$\mathbb{Q}(\sqrt{2})$	$p \equiv 1, 7 \bmod 8$	$p = 2$
$\mathbb{Q}(\sqrt{-2})$	$p \equiv 1, 3 \bmod 8$	$p = 2$
$\mathbb{Q}(\sqrt{3})$	$p \equiv 1, 11 \bmod 12$	$p = 2, 3$
$\mathbb{Q}(\sqrt{-3})$	$p \equiv 1 \bmod 3$	$p = 3$
$\mathbb{Q}(\sqrt{5})$	$p \equiv 1, 4 \bmod 5$	$p = 5$
$\mathbb{Q}(\sqrt{-5})$	$p \equiv 1, 3, 7, 9 \bmod 20$	$p = 2, 5$
$\mathbb{Q}(\sqrt{6})$	$p \equiv 1, 5, 23, 19 \bmod 24$	$p = 2, 3$
$\mathbb{Q}(\sqrt{-6})$	$p \equiv 1, 5, 7, 11 \bmod 24$	$p = 2, 3$
$\mathbb{Q}(\sqrt{-15})$	$p \equiv 1, 2, 4, 8 \bmod 15$	$p = 3, 5$

TABLE 5.5. All fields in which the decomposition of a prime number is determined by $p \bmod 7$

Field L	$[L : \mathbb{Q}]$	totally decomposed prime numbers p	ramified prime numbers p
$\mathbb{Q}(\zeta_7)$	6	$p \equiv 1 \bmod 7$	$p = 7$
$\mathbb{Q}(\zeta_7 + \zeta_7^{-1})$	3	$p \equiv 1, 6 \bmod 7$	$p = 7$
$\mathbb{Q}(\sqrt{-7})$	2	$p \equiv 1, 2, 4 \bmod 7$	$p = 7$
$\mathbb{Q}$	1	all p	none

TABLE 5.6. All fields in which the decomposition of a prime number is determined by $p \bmod 20$

Field L	$[L : \mathbb{Q}]$	totally decomposed prime numbers p	ramified prime numbers p
$\mathbb{Q}(\zeta_{20})$	8	$p \equiv 1 \bmod 20$	$p = 2, 5$
$\mathbb{Q}(\zeta_5)$	4	$p \equiv 1 \bmod 5$	$p = 5$
$\mathbb{Q}(\zeta_{20} + \zeta_{20}^{-1})$	4	$p \equiv 1, 19 \bmod 20$	$p = 2, 5$
$\mathbb{Q}(\sqrt{5}, \sqrt{-1})$	4	$p \equiv 1, 9 \bmod 20$	$p = 2, 5$
$\mathbb{Q}(\sqrt{5})$	2	$p \equiv 1, 4 \bmod 5$	$p = 5$
$\mathbb{Q}(\sqrt{-5})$	2	$p \equiv 1, 3, 7, 9 \bmod 20$	$p = 2, 5$
$\mathbb{Q}(\sqrt{-1})$	2	$p \equiv 1 \bmod 4$	$p = 2$
$\mathbb{Q}$	1	all p	none

From K. Kato, N. Kurokawa, T. Saito, <u>Number Theory 2. Introduction to Class Field Theory</u>, pp. 7-9