Exercise Sheet 11

Properties of Morphisms, Constructible Sets

Exercise 2 is taken from *Algebraic Geometry* by Hartshorne. Exercise 6 is adapted from *Algebraic Geometry I* by Görtz and Wedhorn.

1. Is any open subscheme of any quasicompact scheme X quasicompact? What if X is noetherian?

2. Show by example that a surjective quasi-finite morphism of finite type need not be finite.

3. Prove that any finite morphism $f\colon X \to Y$ is projective.

4. Let $\square$ be one of the properties quasicompact, of finite type, locally of finite type, affine, integral, finite, projective, or quasiprojective.

   (a) Show that a morphism $f\colon X \to Y$ is $\square$ if and only if there exists an open covering $Y = \bigcup_i V_i$ such that $f^{-1}(V_i) \to V_i$ is $\square$ for every i.

   *(b) Consider two morphisms $X \xrightarrow{f} Y \xrightarrow{g} Z$ such that $g \circ f$ is $\square$. In which case does it follow that f is $\square$? Which additional condition on f or g would guarantee that?

5. Let X be a scheme of finite type over a field. Show that any constructible subset of X which contains all closed points of X is equal to X.

6. Let $f\colon X \to Y$ be a surjective morphism of finite type with Y noetherian. Show that a subset C of Y is constructible if and only if $f^{-1}(C)$ is constructible.

*7. Let X be of finite type over a noetherian scheme S. Show that the set of points $s \in S$ where the fiber X_s has a fixed dimension d is constructible.