Carina Ehrensegger

Exercise: down & out put option:

$$C^{put}_{d\&o} = (K - S_T)_+ \mathbb{1}_{\min_{0 \le t \le T} S_t \ge B}$$

where

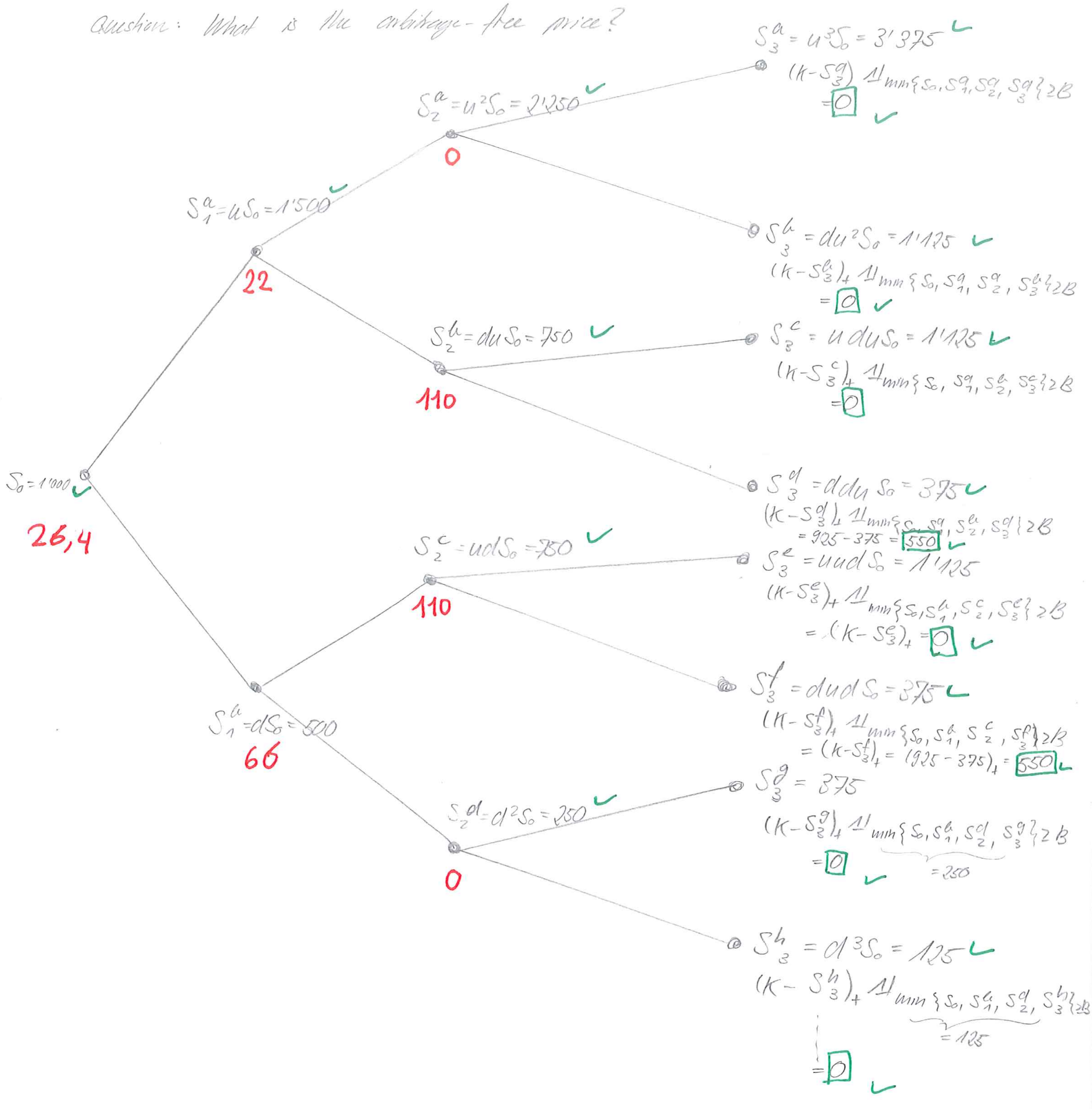
$B = 300$ $\qquad$ $K = 925$

$S_0 = 1'000$ $\qquad$ $e^r = 5/4$

$u = 3/2$

$d = 1/2$ $\qquad$ $p^* = \frac{e^r - d}{u - d} = \frac{5/4 - 1/2}{3/2 - 1/2} = 5/4 - 1/2 = 5/4 - 2/4 = 3/4$

Question: What is the arbitrage-free price?



$$c(S_2^a) = \frac{p^* c(S_3^a) + (1-p^*) c(S_3^b)}{e^r} = \frac{p^* \cdot 0 + (1-p^*) \cdot 0}{e^r} = 0 \checkmark$$

$$c(S_2^b) = \frac{p^* c(S_3^c) + (1-p^*) c(S_3^d)}{e^r} = \frac{p^* \cdot 0 + (1-p^*) \cdot 550}{e^r} = \frac{(1-3/4) \cdot 550}{5/4} = \frac{550}{5} = 110 \checkmark$$

$$c(S_2^c) = \frac{p^* c(S_3^e) + (1-p^*) c(S_3^f)}{e^r} = \frac{p^* \cdot 0 + (1-p^*) \cdot 550}{e^r} = 110 \checkmark$$

$$c(S_2^d) = \frac{p^* c(S_3^g) + (1-p^*) c(S_3^h)}{e^r} = \frac{p^* \cdot 0 + (1-p^*) \cdot 0}{e^r} = 0 \checkmark$$

$$c(S_1^a) = \frac{p^* c(S_2^a) + (1-p^*) c(S_2^b)}{e^r} = \frac{p^* \cdot 0 + (1-p^*) \cdot 110}{e^r} = \frac{(1-3/4) \cdot 110}{5/4} = \frac{110}{5} = 22 \checkmark$$

$$c(S_1^b) = \frac{p^* c(S_2^c) + (1-p^*) c(S_2^d)}{e^r} = \frac{p^* \cdot 110 + (1-p^*) \cdot 0}{e^r} = \frac{3/4 \cdot 110}{5/4} = \frac{3 \cdot 110}{5} = 3 \cdot 20 = 66 \checkmark$$

$$c(S_0) = \frac{p^* c(S_1^a) + (1-p^*) c(S_1^b)}{e^r} = \frac{p^* \cdot 22 + (1-p^*) \cdot 66}{e^r} = \frac{3/4 \cdot 22 + 1/4 \cdot 66}{5/4}$$

$$= \frac{3 \cdot 22 + 1 \cdot 66}{5} = \frac{66 + 66}{5} = \frac{132}{5} = \underline{\underline{26{,}4}} \checkmark$$

arbitrage-free price for C^{put}_{d80}