ETH Zürich
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Introduction to mathematical finance

Solution sheet 13

Solution 13.1

1. First note that $(\xi \bullet X)_T \subset \mathcal{C}$. Therefore, $u_{\mathcal{C}}(x) \geq u(x)$.

   Suppose f^* is a maximizer. Then, since $f^* \in \mathcal{C}$, $f^* = (\xi^* \bullet X)_T - Y$ for some $\xi^* \in \Xi$ and $Y \geq 0$, and
   $$u_{\mathcal{C}}(x) = E\left[U(x + (\xi^* \bullet X)_T - Y)\right].$$
   Since U is strictly increasing, Y must be identically zero. Hence, $u_{\mathcal{C}}(x) = u(x)$, and the optimizer f^* corresponds to an optimizer ξ^* for the first problem.

   On the other hand, if ξ^* is an optimizer of the first problem, then $f^* = (\xi^* \bullet X)_T$ must optimize the second, for otherwise there would exist a strictly better f', and by the argument above also a strictly better ξ', violating the assumption that ξ^* is an optimizer.

2. Since Ω is finite, every f is bounded from below by $\min_\omega f$. Therefore, by Theorem II.7.2,
   $$f \in \mathcal{C} \quad \Longleftrightarrow \quad E_Q[f] \leq 0, \quad \forall Q \in \mathbb{P}_e.$$
   We need to extend this statement to $\mathbb{P}_a$. If $E_Q[f] \leq 0$ for all $Q \in \mathbb{P}_a$, the desired implication holds trivially. On the other hand, suppose $f \in \mathcal{C}$. Then $E_Q[f] \leq 0$ for all EMMs Q. Thus,
   $$\sup_{Q \in \mathbb{P}_e} E_Q[f] \leq 0,$$
   and, by Exercise 3.1,
   $$\sup_{Q \in \mathbb{P}_a} E_Q[f] \leq 0.$$
   This is what we wanted to show.