Problem 1. Consider the curve

$$K = \{[X : Y : Z] \mid X^3Y + Y^3Z + Z^3X = 0\} \subset \mathbb{CP}^2,$$

the plane

$$H = \{[X : Y : Z] \mid X + Y + Z = 0\},$$

and the rational map

$$\begin{aligned} \varphi : K &\dashrightarrow H \\ [X : Y : Z] &\mapsto [X^3Y : Y^3Z : Z^3X] \end{aligned}$$

a) Prove explicitly that φ extends to a regular map on all of K by giving a description near the points at which it is not defined.

b) Now look at the resulting map $K \to H$. What is its degree? Compute the branch points and the ramification indices. Check that the Riemann–Hurwitz formula holds.

Problem 2. Let $\mathbb{F}_q$ be a finite field. Use your arguments in Problem 1 to count

$$\#\{[X : Y : Z] \in \mathbb{P}^2(\mathbb{F}_q) \mid X^3Y + Y^3Z + Z^3X = 0\}$$

in the case

$$q \not\equiv 1 \pmod 7.$$