# Serie 11

1. Consider the Laplace equation $\Delta u = 0$ in the domain $0 < x, y < \pi$ with the boundary condition
$$u_y(x, \pi) = x^2 - a, \quad u_x(0, y) = u_x(\pi, y) = u_y(x, 0) = 0.$$
Find all the values of the parameter $a$ for which the problem is solvable.

2. a) Find nontrivial solutions of special form $\omega(x, y) = X(x)Y(y)$ of the equation
$$\begin{aligned} &\omega_{xx} + \omega_{yy} = 0 && 0 < x < 2\pi,\ -1 < y < 1, \\ &\omega_x(0, y) = \omega_x(2\pi, y) = 0 && -1 < y < 1. \end{aligned}$$
**Hint.** *Read section 7.7.1. This problem can be solved by the similar method used there.*

   b) Solve the problem
$$\begin{aligned} &u_{xx} + u_{yy} = 0 && 0 < x < 2\pi,\ -1 < y < 1, \\ &u(x, -1) = 0, \quad u(x, 1) = 1 + \cos 2x && 0 \leqslant x \leqslant 2\pi, \\ &u_x(0, y) = u_x(2\pi, y) = 0 && -1 < y < 1. \end{aligned}$$
**Hint.** *Write the solution as an expansion of solutions of special form found in a).*

3. Consider the Newmann problem
$$\begin{aligned} &\Delta u(x, y) = 0, && (x, y) \in [0, \pi] \times [0, \pi], \\ &u_x(0, y) = 0,\ u_x(\pi, y) = \sin y, && y \in [0, \pi], \\ &u_y(x, 0) = 0,\ u_y(x, \pi) = -\sin x, && x \in [0, \pi] \end{aligned}$$
In order to divide the problem into two subproblems, find $a$ such that
$$\begin{aligned} &\Delta v_1(x, y) = 0, && (x, y) \in [0, \pi] \times [0, \pi], \\ &(v_1)_x(0, y) = a(x^2 - y^2)_x, && \\ &(v_1)_x(\pi, y) = \sin y + a(x^2 - y^2)_x, && y \in [0, \pi], \\ &(v_1)_y(x, 0) = 0,\ (v_1)_y(x, \pi) = 0, && x \in [0, \pi] \end{aligned}$$

and

$$\begin{aligned}
&\Delta v_2(x,y) = 0, && (x,y) \in [0,\pi] \times [0,\pi],\\
&(v_2)_x(0,y) = 0,\ (v_2)_x(\pi,y) = 0, && y \in [0,\pi],\\
&(v_2)_y(x,0) = a(x^2 - y^2)_y, &&\\
&(v_2)_y(x,\pi) = -\sin x + a(x^2 - y^2)_y, && x \in [0,\pi]
\end{aligned}$$

are both solvable.

**Remark.** *This is an example of the problem in Remark 7.25 in page 194 of [PR].*

# References


[PR] Y. Pinchover, J. Rubinstein, An introduction to Partial Differential Equations, Cambridge University Press(12. Mai 2005).