Serie 2

Recall: Given the PDE

$$a(x, y, u)u_x + b(x, y, u)u_y = c(x, y, u),$$

the system of ODEs

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}x(t,s) = a(x(t,s), y(t,s), u(t,s)) \\ \dfrac{\mathrm{d}}{\mathrm{d}t}y(t,s) = b(x(t,s), y(t,s), u(t,s)) \\ \dfrac{\mathrm{d}}{\mathrm{d}t}u(t,s) = c(x(t,s), y(t,s), u(t,s)) \end{cases}$$

is called the characteristic equations and the solutions are called characteristic curves.The curves $(x(t,s), y(t,s))$ are the projections of the characteristic curves on the (x, y) plane.

Solve the following equations by the method of characteristics:

1) $u_x + u_y = 2u + 1, \quad u(x, 0) = 0$

2) $xu_x + (x + y)u_y = 1, \quad u(1, y) = y$

3) $u_x + 2u_y = e^u, \quad u(0, y) = 1.$

4) $xu_x + yu_y = -u, \quad u(\cos s, \sin s) = 1, \quad 0 \leqslant s \leqslant \pi.$