Serie 8

1. (Nonhomogeneous heat equation, with Newmann boundary conditions) Solve the problem

$$\begin{aligned} &u_t - u_{xx} = 2t + 15\cos 2x && 0 < x < \frac{\pi}{2}, \quad t > 0, \\ &u_x(0,t) = u_x(\frac{\pi}{2}, t) = 0 && t \geqslant 0, \\ &u(x,0) = 1 + 3\cos 4x && 0 \leqslant x \leqslant \frac{\pi}{2}. \end{aligned}$$

Hint. *It is similar to example 6.45 in [PR]. Write the solution $u(x,t)$ as the eigenfunction expansion. Notice the boundarys here are Newmann boundary conditions.*

2. (Nonhomogeneous heat equations with nonhomogeneous Newmann boundary conditions) Solve the problem

$$\begin{aligned} &u_t - u_{xx} = 1 + x\cos t && 0 < x < 1, \quad t > 0, \\ &u_x(0,t) = u_x(1,t) = \sin t && t \geqslant 0, \\ &u(x,0) = 1 + \cos(2\pi x) && 0 \leqslant x \leqslant 1. \end{aligned}$$

Hint. *The general solution of nonhomogeneous boundary conditions is given in section* 6.6 *of [PR]. Construct a suitable auxiliary function $w(x,t)$ satisfying the nonhomogeneous boundary conditions. Then set $v(x,t) = u(x,t) - w(x,t)$ and solve the equations satisfied by v. To solve the equation of v, we again write v as the eigenfunction expansion.*

3. (Nonhomogeneous heat equations with Dirichlet boundary conditions) Solve the problem

$$\begin{aligned} &u_t - u_{xx} + 4u = 0 && 0 < x < \pi, \quad t > 0, \\ &u(0,t) = u(\pi,t) = 0 && t \geqslant 0, \\ &u(x,0) = \sin 2x && 0 \leqslant x \leqslant \pi. \end{aligned}$$

Hint. *Write the solution $u(x,t)$ as the eigenfunction expansions*

4. (Nonhomogeneous heat equations with nonhomogeneous Dirichlet boundary conditions)

$$\begin{aligned} &u_t - u_{xx} + 4u = \frac{17}{4}\sin\frac{x}{2} && 0 < x < \pi, \quad t > 0, \\ &u(0,t) = 0, u(\pi,t) = 1 && t \geqslant 0, \\ &u(x,0) = \sin\frac{x}{2} + \sin 2x && 0 \leqslant x \leqslant \pi. \end{aligned}$$

Hint. *Consider the auxiliary function* $w(x,t) = \sin \frac{x}{2}$.

References

[PR] Y. Pinchover, J. Rubinstein, An introduction to Partial Differential Equations, Cambridge University Press(12. Mai 2005).