Serie 9

1. A harmonic function of the form

$$P_n(x,y) = \sum_{i+j=n} a_{i,j} x^i y^j$$

is called a *homogeneous harmonic polynomial* of degree n. Denote the space of homogeneous harmonic polynomials of degree n by V_n. What is the dimension of V_n? What is the space V_n?

Hint. *Use the polar form of the Laplace equation (see formula (7.11) on page 177 in [PR]).*

2. Consider the Dirichlet problem for *reduced Helmholtz equation* in a bounded planar domain D

$$\begin{aligned} &\Delta u(x,y) - ku(x,y) = 0 && (x,y) \in D, \\ &u(x,y) = g(x,y) && (x,y) \in \partial D, \end{aligned}$$

where k is a positive constant. Prove the problem has at most one solution in $C^2(D) \cap C(\overline{D})$.

Hint. *Applying the similar ideas in theorem 7.5 and theorem 7.12 in [PR].*

3. Let u be a harmonic function in a planar domain D and (x_0, y_0) be a point in D. Assume that B_R is a disk of radius R centered at (x_0, y_0), fully contained in D. Then the value of u at (x_0, y_0) is the average of the values of u in the disk B_R:

$$u(x_0, y_0) = \frac{1}{\pi R^2} \int_{B_R} u(x,y) \mathrm{d}x \mathrm{d}y.$$

References

[PR] Y. Pinchover, J. Rubinstein, An introduction to Partial Differential Equations, Cambridge University Press(12. Mai 2005).