

Endterm exam

Num. CSE, D-INFK/D-MATH

HS 2016

Prof. R. Hiptmair

A

Family name		Grade
First name		
Department		
Legi Nr.		
Mark if you did <i>not</i> pass the midterm:		
Date	23.12.2016	

1	2	3	4	5	Total
4	6	6	4	4	24

- **Keep only writing material and Legi on the table.**
- Keep mobile phones, tablets, smartwatches, etc. **turned off** in your bag.
- Fill in this cover sheet first.
- **Turn the cover sheet only when instructed to do so.**
- **Read the rules on the next page carefully.**
- Do not write with red/green color or with pencil.
- **Make sure to hand in every sheet.**
- **Duration: 30 min.**
- Additional material: none.

Wish you much success!

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Rules:

- Motivation for the answers is **not** necessary. Remarks and computations have **no** influence on the total number of points.
- Wrong answers (for multiple choice problems) give negative points. The minimum number of points for each problem is 0.
- All notes outside the predefined boxes will not be considered.
- Each multiple choice box has one and only one correct answer.
- If required, write your solution in the predefined box:

your text here

- Any unclear marking will be considered an error.

1. *LT-FIR* [4 P.]

A linear time-invariant channel (*LT-FIR*) with finite impulse response

$$(\dots, 0, h_0, \dots, h_m, 0, \dots), \quad h_i \in \mathbb{R}, \quad m \in \mathbb{N},$$

is fed with an n -periodic signal $(x_j)_{j \in \mathbb{Z}}$ ($n \in \mathbb{N}$). The output signal $(y_j)_{j \in \mathbb{Z}}$ will be n -periodic again and is given by the formula:

$$y_j = \sum_{l=0}^{n-1} p_l x_{j-l}, \quad j = 0, \dots, n-1,$$

where the coefficients $p_l, l = 0, \dots, n-1$, may depend on h_i, m and n .

Which of the following formulas for $p_l, l \in \{0, \dots, n-1\}$, is/are correct: [+1P correct, -1P wrong, min: 0P]

- $p_l = \sum_{k \in \mathbb{Z}} h_{l+km}$ correct wrong
- $p_l = \sum_{k \in \mathbb{Z}} h_{l+kn}$ correct wrong
- $p_l = \sum_{k \in \mathbb{Z}} h_{l-kn}$ correct wrong
- $p_l = \sum_{k=0}^{\infty} h_{l+kn}$ correct wrong

Scratch space (not evaluated):

2. Discrete convolution [6 P.]

A discrete convolution of two vectors $\mathbf{p}, \mathbf{x} \in \mathbb{R}^n$ is defined as:

$$(\mathbf{p} * \mathbf{x})_j := \sum_{l=0}^j (\mathbf{x})_l (\mathbf{p})_{j-l}, \quad j = 0, \dots, n-1.$$

Note that vector indices start from 0!

For which of the following matrices $\mathbf{A} \in \mathbb{R}^{n,n}$ does there exist a vector $\mathbf{p} \in \mathbb{R}^n$ such that

$$\mathbf{p} * \mathbf{x} = \mathbf{A}\mathbf{x}, \quad \forall \mathbf{x} \in \mathbb{R}^n \quad ?$$

If such \mathbf{p} exists, specify its components. [+1P correct "p exists"?, -1P wrong "p exists"?, +3P correct p entries if p exists, min: 0P]

(a)

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & & & & \\ 1 & 2 & & & & \\ & \ddots & \ddots & & & \\ & & \ddots & \ddots & & \\ & & & \ddots & \ddots & 0 \\ & & & & 1 & 2 \end{bmatrix}$$

\mathbf{p} exists? true false

If \mathbf{p} exists, $\mathbf{p} = (2, 1, \dots, 0, \dots)^\top$

(b)

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & & & & \\ 0 & 2 & -1 & & & \\ & & \ddots & \ddots & & \\ & & & \ddots & \ddots & \\ -1 & & & & 0 & 2 \end{bmatrix}$$

\mathbf{p} exists? true false

If \mathbf{p} exists, $\mathbf{p} =$

(c)

$$\mathbf{A} = \begin{bmatrix} 1 & & & & 0 \\ 2 & 2 & & & \\ 3 & 3 & 3 & & \\ \vdots & & & \ddots & \\ n & n & n & n & n \end{bmatrix}$$

p exists? true false

If **p** exists, **p** =

3. Interpolation error [6 P.]

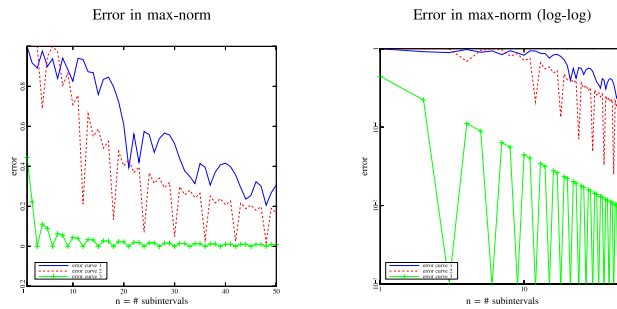
The following plots show the maximum norms of the interpolation errors of piecewise linear interpolation of the three functions on $[0, 1]$

(a) $f(t) = |\sin(6\pi t)|$

(b) $f(t) = |x - \frac{1}{3}|$

(c) $f(t) = |\sin(6\pi t^2)|$

on the equidistant node set $\mathcal{J}_n = \{\frac{j}{n} : j = 0, \dots, n\}$, with $n \in \mathbb{N}$.



Assign the error curves to the individual functions.

[+2P correct assignment, -1P wrong assignment, min: 0P]

Function $f(t)$ error curve 1 error curve 2 error curve 3

$ \sin(6\pi t) $	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
$ x - \frac{1}{3} $	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
$ \sin(6\pi t^2) $	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>

4. Newton method [4 P.]

Given a regular matrix $\mathbf{A} \in \mathbb{R}^{n,n}$, which of the following formulas describe the Newton iteration for solving $F(\mathbf{X}) = \mathbf{0}_{n,n}$ with

$$F : \mathbb{R}^{n,n} \rightarrow \mathbb{R}^{n,n}, \quad F(\mathbf{X}) := \mathbf{A}\mathbf{X} - \mathbf{I}_{n,n} \quad ?$$

[+1P correct, -1P wrong, min: 0P]

- $\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + \mathbf{A}^{-1}(\mathbf{I} - \mathbf{A}\mathbf{X}^{(k)})$ true false
- $\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)}(\mathbf{I} - \mathbf{A}\mathbf{X}^{(k)})$ true false
- $\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + \mathbf{A}\mathbf{X}^{(k)} - \mathbf{I}$ true false
- $\mathbf{X}^{(k+1)} = \mathbf{A}^{-1}$ true false

Scratch space (not evaluated):

5. Convergence [4 P.]

Which of the following statements hold true for a linearly convergent sequence $(\mathbf{x}^{(k)})_{k \in \mathbb{N}_0}$, $\mathbf{x}^{(k)} \in \mathbb{R}^n$, with limit $\mathbf{x}^* = \lim_{k \rightarrow \infty} \mathbf{x}^{(k)}$? [+1P correct, -1P wrong, min: 0P]

- $\exists L \in [0, 1[: \|\mathbf{x}^{(k)} - \mathbf{x}^*\| \leq L^k \|\mathbf{x}^{(0)} - \mathbf{x}^*\|$ true false
- $\exists L \in [0, 1[: \|\mathbf{x}^{(k+1)}\| \leq L \|\mathbf{x}^{(k)}\|$ true false
- $\exists L \in [0, 1[: \|\mathbf{x}^{(k+1)} - \mathbf{x}^*\| \leq L \|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\|$ true false
- $\exists C \geq 0 : \|\mathbf{x}^{(k+1)} - \mathbf{x}^*\| \leq C \|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\|$ true false

$\|\cdot\|$ is a vector norm on \mathbb{R}^n .

Scratch space (not evaluated):