Worksheet 2: Conformal symmetry and the Virasoro algebra

Goal: To motivate the definition of Vertex Operator Algebras, we need to understand a few facts about conformal transformations and the Virasoro algebra. This worksheet will guide you through this. You will work on it at home, and then discuss your answers in class on November 1. Thomas Gemünden will moderate the discussion.

Reference: Chapters 4 and 5 of the book *A Mathematical Introduction to Conformal Field Theory*. You can find an electronic copy through the ETH library at:
`https://link.springer.com/book/10.1007%2F978-3-540-68628-6`

Tasks:

- Work through the questions at home and try to answer them. Be ready to ask questions during class about the problems that you could not solve, and volunteer to explain the ones that you could.

Questions

1. (Physics:) In quantum mechanics, what is a physical state? If we have a symmetry group G of a system, why do we care about its projective representations?

2. (Mathematics:) If we are interested in projective representations, why do we care about central extensions?

3. Explain Lemma 4.5 and Remark 4.7 in section 4.

4. What is the Witt algebra? How is it related to conformal transformations on $\mathbb{C}$? (Section 5.2)

5. What is the Virasoro algebra? Prove theorem 5.1. (Section 5.3)