Worksheet 1: Basics of modular forms

Goal: We will need some basic knowledge about modular forms later on in the lecture. This worksheet will help you get up to speed. You will work on it at home, and then discuss your answers in class on October 25. Paloma Bengoechea will moderate the discussion.

Reference: Sections 1 and 2 of part 1 (*Elliptic Modular Forms and their Applications*) of the book *The 1–2–3 of Modular Forms*. You can find an electronic copy through the ETH library at:
`https://link.springer.com/book/10.1007/978-3-540-74119-0`

Tasks:

- If you don't know modular forms, work through the questions at home and try to answer them. Be ready to ask questions during class.
- If you already know the topic, work through the worksheet at home to refresh your memory. During class, volunteer to explain the solutions to your classmates.

Questions

1. What are modular forms? What is their Fourier expansion? (Section 1.1)
2. How is the modular group generated? Prove Proposition 1. (1.2)
3. What is the dimension of the space of modular forms? Prove Proposition 2 and Corollary 1. (1.3)
4. How are Eisenstein series constructed? Explain Proposition 4 (2.1)
5. Compute the Fourier expansion of the Eisenstein series. (2.2)
6. Prove that the discriminant function Δ is a modular form of weight 12. (Proposition 7 in 2.4)