9.1. Minkowski functional

Define the set $\Upsilon \subset X^*$ by using p and prove the two inclusions. By showing $p(x) < 1$ for $x \in Q$, one inclusion follows directly. Prove the other inclusion indirectly.

(The set $\Upsilon \subset X^*$ is not required to be open.)

9.2. Extremal points

(a) Show that E is a subset of the boundary ∂K. Derive a contradiction to $E \subset \partial K$ under the assumption that E is not closed.

(b) Draw a picture.

9.3. Extremal subsets

(a) $K \setminus M$ being not convex contradicts the definition of extremal subset. Notice where convexity of K is used.

(b) Have you tried intervals?

(c) If $y \in K$ is an extremal point of K, then $\{y\} \subset K$ is an extremal subset of K.

9.4. Weak sequential continuity of linear operators

Prove (ii) $\Rightarrow$ (i) by contradiction and use that weakly convergent sequences must be bounded (Satz 4.6.1).

9.5. Weak convergence in finite dimensions

Recall that all norms are equivalent in finite dimensions.

9.6. Weak convergence in Hilbert spaces

(a) $x_n \xrightarrow{\mathrm{w}} x$ implies $(x, x_n)_H \to (x, x)_H$.

(b) Weakly convergent sequences are bounded.

(c) Recall Bessel's inequality.

(d) Use (c).

(e) Use (c).