ETH Zurich HS 2017
D-MATH, D-PHYS
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Coordinator
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Probability Theory

Exercise Sheet 8

Exercise 8.1 Let $S, T : \Omega \to \mathbb{N} \cup \{\infty\}$ be $\mathcal{F}_n$-stopping times. Prove or provide a counter example disproving the following statements:

(a) $S - 1$ is a stopping time.

(b) $S + 1$ is a stopping time.

(c) $S \wedge T$ is a stopping time.

(d) $S \vee T$ is a stopping time.

(e) $S + T$ is a stopping time.

Exercise 8.2 Let $n \geq 2$, and let $X_1, \ldots, X_n$ be i.i.d. random variables defined on a probability space $(\Omega, \mathcal{A}, P)$.

(a) Show that for every Borel function $g : \mathbb{R}^n \to \mathbb{R}$ with $E[|g(X_1, \ldots, X_n)|] < \infty$ and any permutation π of $\{1, \ldots, n\}$,

$$E[g(X_1, \ldots, X_n)] = E[g(X_{\pi(1)}, \ldots, X_{\pi(n)})].$$

(b) Set $S := X_1 + \ldots + X_n$ and assume that X_1 is integrable. Find a representation of $E[X_1|S]$ as a function of S. *Hint:* First show that $E[X_1|S] = E[X_2|S]$ P-a.s.

Exercise 8.3 Let Y_n, $n \geq 0$ be i.i.d. with $P[Y_0 = 1] = p$ and $P[Y_0 = 0] = 1 - p$ for some $p \in (0, 1)$. Let $\mathcal{F}_n := \sigma(Y_0, \ldots, Y_n)$ for $n \geq 0$ and define

$$T := \inf\{n \geq 0 \mid Y_n = 1\}.$$

Determine the Doob decomposition of $X_n := 1_{\{T \leq n\}}$, $n \geq 0$.

Hint: First check that X_n is an $\mathcal{F}_n$-submartingale.

Submission deadline: 13:15, Nov 21.

Location: During exercise class or in the tray outside of HG E 65.

Office hours (Präsenz): Mon. and Thu., 12:00-13:00 in HG G 32.6.

Class assignment:

Students	Time & Date	Room	Assistant
An-Gr	Tue 13-14	HG F 26.5	Yilin Wang
He-Lang	Tue 13-14	ML H 41.1	Angelo Abächerli
Lanz-Sa	Tue 14-15	HG F 26.5	Vincenzo Ignazio
Sch-Zh	Tue 14-15	ML H 41.1	Lukas Gonon