ETH Zurich HS 2017
D-MATH, D-PHYS
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Coordinator
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Probability Theory

Self evaluation quiz, November 14

Number:

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1. (a) State the Three Series Theorem.
   (b) Suppose that $X_k = Z_k/k^{1/4}$, for $k \geq 1$, where Z_k are i.i.d. random variables with $P(Z_k = 1) = P(Z_k = -1) = 1/4$ and $P(Z_k = 0) = 1/2$. Discuss the convergence properties of the random series $\sum_{k\geq 1} X_k$.

2. Suppose that X_k, $k \geq 1$ are i.i.d. random variables with symmetric stable distribution with parameters $0 < \alpha < 2$ and $c > 0$.
   (a) What is the law of $Z_n = \frac{1}{\sqrt{n}}(X_1 + \cdots + X_n)$?
   (b) Does Z_n converge in distribution? Justify your answer.

3. (a) State the Kolmogorov 0-1 law.
   (b) Give an example where it applies.

4. Assume that $X \in L^2(\Omega, \mathcal{A}, P)$ and $A \in \mathcal{A}$ is such that $0 < P(A) < 1$. Let $\mathcal{F} = \{\emptyset, A, A^c, \Omega\}$.
   (a) Compute $E[X|\mathcal{F}]$.
   (b) If $Y = E[(X - E[X|\mathcal{F}])^2|\mathcal{F}]$ is the conditional variance of X given $\mathcal{F}$, express $E[Y]$ in terms of the variance of $Z = E[X|\mathcal{F}]$ and the variance of X, justify your answer.