Non-Life Insurance: Mathematics and Statistics

Exercise sheet 12

Exercise 12.1 Chain-Ladder and Bornhuetter-Ferguson
Set $I = J + 1 = 10$. Assume that for the cumulative payments $C_{i,j}$ with $1 \leq i \leq 10$ and $0 \leq j \leq 9$ we have observations

$$\mathcal{D}_{10} = \{C_{i,j} \,|\, i + j \leq 10,\, 1 \leq i \leq 10,\, 0 \leq j \leq 9\}$$

given by the following claims development triangle:

accident	development year j									
year i	0	1	2	3	4	5	6	7	8	9
1	5'946'975	9'668'212	10'563'929	10'771'690	10'978'394	11'040'518	11'106'331	11'121'181	11'132'310	11'148'124
2	6'346'756	9'593'162	10'316'383	10'468'180	10'536'004	10'572'608	10'625'360	10'636'546	10'648'192	
3	6'269'090	9'245'313	10'092'366	10'355'134	10'507'837	10'573'282	10'626'827	10'635'751		
4	5'863'015	8'546'239	9'268'771	9'459'424	9'592'399	9'680'740	9'724'068			
5	5'778'885	8'524'114	9'178'009	9'451'404	9'681'692	9'786'916				
6	6'1847'93	9'013'132	9'585'897	9'830'796	9'935'753					
7	5'600'184	8'493'391	9'056'505	9'282'022						
8	5'288'066	7'728'169	8'256'211							
9	5'290'793	7'648'729								
10	5'675'568									

Note that you can download this data set from

https://people.math.ethz.ch/~wueth/exercises2.html

by clicking on "Data to the Examples".

(a) Predict the lower triangle

$$\mathcal{D}_{10}^c = \{C_{i,j} \,|\, i + j > 10,\, 1 \leq i \leq 10,\, 0 \leq j \leq 9\}$$

using the chain-ladder (CL) method. Calculate the CL reserves $\widehat{\mathcal{R}}_i^{CL}$ for all accident years $i \in \{1, \ldots, 10\}$.

(b) Assume we have prior informations $\widehat{\mu}_1, \ldots, \widehat{\mu}_{10}$ for the total expected ultimate claims $\mathbb{E}[C_{1,J}], \ldots, \mathbb{E}[C_{10,J}]$ given by

accident year	1	2	3	4	5
prior information	11'653'101	11'367'306	10'962'965	10'616'762	11'044'881
accident year	**6**	**7**	**8**	**9**	**10**
prior information	11'480'700	11'413'572	11'126'527	10'986'548	11'618'437

Calculate the claim reserves $\widehat{\mathcal{R}}_i^{BF}$ for all accident years $i \in \{1, \ldots, 10\}$ using the Bornhuetter-Ferguson (BF) method.

(c) Explain why in this example we have

$$\widehat{\mathcal{R}}_i^{CL} < \widehat{\mathcal{R}}_i^{BF},$$

for all accident years $i \in \{2, \ldots, 10\}$.

Exercise 12.2 Mack's Formula and Merz-Wüthrich (MW) Formula (R Exercise)
Consider the identical setup as in Exercise 12.1. Write an R-Code using the R-package `ChainLadder` in order to calculate the conditional mean square error of prediction

$$\mathrm{msep}^{\mathrm{Mack}}_{C_{i,J}|\mathcal{D}_I}\left(\widehat{C}^{CL}_{i,J}\right),$$

for all $i \in \{1, \dots, 10\}$, given in formula (9.21) of the lecture notes, as well as

$$\mathrm{msep}^{\mathrm{Mack}}_{\sum_{i=1}^I C_{i,J}|\mathcal{D}_I}\left(\sum_{i=1}^I \widehat{C}^{CL}_{i,J}\right),$$

for aggregated accident years, given in formula (9.22) of the lecture notes. Interpret the square-rooted conditional mean square errors of prediction relative to the claims reserves calculated in Exercise 12.1, (a). Moreover, determine

$$\mathrm{msep}^{\mathrm{MW}}_{\mathrm{CDR}_{i,I+1}|\mathcal{D}_I}(0),$$

for all $i \in \{1, \dots, 10\}$, given in formula (9.34) of the lecture notes as well as

$$\mathrm{msep}^{\mathrm{MW}}_{\sum_{i=1}^I \mathrm{CDR}_{i,I+1}|\mathcal{D}_I}(0),$$

for aggregated accident years, given in formula (9.35) of the lecture notes.

Exercise 12.3 Conditional MSEP and Claims Development Result
In the first part of this exercise we show that the conditional mean square error of prediction can be decoupled into the process uncertainty and the parameter estimation error. In the second part we show that the claims development results are uncorrelated.

(a) Let $\mathcal{D}$ be a σ-algebra, $\widehat{X}$ a $\mathcal{D}$-measurable predictor for the random variable X and both $\widehat{X}$ and X square-integrable. Show that

$$\mathrm{msep}_{X|\mathcal{D}}\left(\widehat{X}\right) = \mathbb{E}\left[\left(X - \widehat{X}\right)^2 \middle| \mathcal{D}\right] = \mathrm{Var}(X \mid \mathcal{D}) + \left(\mathbb{E}[X \mid \mathcal{D}] - \widehat{X}\right)^2 \quad \text{a.s.}$$

(b) Suppose that we are in the framework of Chapter 9.4.1 of the lecture notes. Let $t_1 < t_2 \in \mathbb{N}$ such that $t_1 \geq I$ and $i > t_2 - J$ and assume that $C_{i,J}$ has finite second moment. Show that

$$\mathrm{Cov}\left(\mathrm{CDR}_{i,t_1+1}, \mathrm{CDR}_{i,t_2+1}\right) = 0.$$