Non-Life Insurance: Mathematics and Statistics

Exercise sheet 3

Exercise 3.1 No-Claims Bonus
An insurance company decides to offer a no-claims bonus to good car drivers, namely

- a 10% discount after three years of no claim, and
- a 20% discount after six years of no claim.

If the base premium is defined as the expected claim amount, how does it need to be adjusted so that this no-claims bonus can be financed? For simplicity, we consider one car driver who has been insured for at least six years. Answer the question in the following two situations:

(a) The claim counts of the individual years of the considered car driver are independent, identically Poisson distributed random variables with frequency parameter $\lambda = 0.2$.

(b) Suppose Θ follows a gamma distribution with shape parameter $\gamma = 1$ and scale parameter $c = 1$, i.e. the density f_Θ of Θ is given by

$$f_\Theta(x) = \begin{cases} \exp\{-x\} & \text{if } x \geq 0, \\ 0 & \text{else.} \end{cases}$$

Now, conditionally given Θ, the claim counts of the individual years of the considered car driver are independent, identically Poisson distributed random variables with frequency parameter $\Theta\lambda$, where $\lambda = 0.2$ as above.

Exercise 3.2 Central Limit Theorem
Let n be the number of claims and $Y_1, \ldots, Y_n$ the corresponding claim sizes, where we assume that $Y_1, \ldots, Y_n$ are independent, identically distributed random variables with expectation $\mathbb{E}[Y_1] = \mu$ and coefficient of variation $\mathrm{Vco}(Y_1) = 4$. Use the Central Limit Theorem to determine an approximate minimum number of claims such that with probability of at least 95% the deviation of the empirical mean $\frac{1}{n}\sum_{i=1}^n Y_i$ from μ is less than 1%.

Exercise 3.3 Compound Binomial Distribution
Assume $S \sim \mathrm{CompBinom}(v, p, G)$ for given $v \in \mathbb{N}$, $p \in (0,1)$ and individual claim size distribution G. Let $M > 0$ such that $G(M) \in (0,1)$. Define the compound distribution of claims Y_i exceeding threshold M by

$$S_{\mathrm{lc}} = \sum_{i=1}^N Y_i \, 1_{\{Y_i > M\}}.$$

Show that $S_{\mathrm{lc}} \sim \mathrm{CompBinom}(v, p[1 - G(M)], G_{\mathrm{lc}})$, where the large claims size distribution function G_{lc} satisfies $G_{\mathrm{lc}}(y) = \mathbb{P}(Y_1 \leq y | Y_1 > M)$.