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Non-Life Insurance: Mathematics and Statistics

Exercise sheet 4

Exercise 4.1 Poisson Model and Negative-Binomial Model
Suppose that we are given the following claim count data of ten years:

t	1	2	3	4	5	6	7	8	9	10
N_t	1'000	997	985	989	1'056	1'070	994	986	1'093	1'054
v_t	10'000	10'000	10'000	10'000	10'000	10'000	10'000	10'000	10'000	10'000

Table 1: Observed claims counts N_t and corresponding volumes v_t.

(a) Estimate the claims frequency parameter $\lambda > 0$ of the Poisson model and calculate an estimated, roughly 70%-confidence interval for λ. What do you observe?

(b) Perform a χ^2-goodness-of-fit test at the significance level of 5% to test the null hypothesis of having Poisson distributions.

(c) Estimate the claims frequency parameter $\lambda > 0$ and the dispersion parameter $\gamma > 0$ of the negative-binomial model and calculate an estimated, roughly 70%-confidence interval for λ. What do you observe?

Exercise 4.2 Compound Poisson Distribution
For the total claim amount S of an insurance company we assume $S \sim \text{CompPoi}(\lambda v, G)$, where $\lambda = 0.06$, $v = 10$ and for a random variable Y with distribution function G we have

k	100	300	500	6'000	100'000	500'000	2'000'000	5'000'000	10'000'000
$\mathbb{P}[Y = k]$	3/20	4/20	3/20	2/15	2/15	1/15	1/12	1/24	1/24

Table 2: Distribution of $Y \sim G$.

Suppose that the insurance company wants to distinguish between

- small claims: claim size $\leq$ 1'000,
- medium claims: 1'000 $<$ claim size $\leq$ 1'000'000 and
- large claims: claim size $>$ 1'000'000.

Let S_{sc}, S_{mc} and S_{lc} be the total claims in the small claims layer, in the medium claims layer and in the large claims layer, respectively.

(a) Give definitions of S_{sc}, S_{mc} and S_{lc} in terms of mathematical formulas.

(b) Determine the distributions of S_{sc}, S_{mc} and S_{lc}.

(c) What is the dependence structure between S_{sc}, S_{mc} and S_{lc}?

(d) Calculate $\mathbb{E}[S_{\text{sc}}]$, $\mathbb{E}[S_{\text{mc}}]$ and $\mathbb{E}[S_{\text{lc}}]$ as well as $\text{Var}(S_{\text{sc}})$, $\text{Var}(S_{\text{mc}})$ and $\text{Var}(S_{\text{lc}})$. Use these values to calculate $\mathbb{E}[S]$ and $\text{Var}(S)$.

(e) Calculate the probability that the total claim in the large claims layer exceeds 5 millions.

Exercise 4.3 Method of Moments

We assume that the independent claim sizes $Y_1, \ldots, Y_8$ all follow a Gamma distribution with the same unknown shape parameter $\gamma > 0$ and the same unknown scale parameter $c > 0$ and that we have the following observations for $Y_1, \ldots, Y_8$:

$$x_1 = 7, \qquad x_2 = 8, \qquad x_3 = 10, \qquad x_4 = 9, \qquad x_5 = 5, \qquad x_6 = 11, \qquad x_7 = 6, \qquad x_8 = 8.$$

Calculate the method of moments estimates of γ and c.