Non-Life Insurance: Mathematics and Statistics

Exercise sheet 4

Exercise 4.1 Poisson Model and Negative-Binomial Model

Suppose that we are given the following claim count data of ten years:

<table>
<thead>
<tr>
<th>$t$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_t$</td>
<td>1'000</td>
<td>997</td>
<td>985</td>
<td>989</td>
<td>1'056</td>
<td>1'070</td>
<td>994</td>
<td>986</td>
<td>1'093</td>
<td>1'054</td>
</tr>
<tr>
<td>$v_t$</td>
<td>10'000</td>
<td>10'000</td>
<td>10'000</td>
<td>10'000</td>
<td>10'000</td>
<td>10'000</td>
<td>10'000</td>
<td>10'000</td>
<td>10'000</td>
<td>10'000</td>
</tr>
</tbody>
</table>

Table 1: Observed claims counts $N_t$ and corresponding volumes $v_t$.

(a) Estimate the claims frequency parameter $\lambda > 0$ of the Poisson model and calculate an estimated, roughly 70%-confidence interval for $\lambda$. What do you observe?

(b) Perform a $\chi^2$-goodness-of-fit test at the significance level of 5% to test the null hypothesis of having Poisson distributions.

(c) Estimate the claims frequency parameter $\lambda > 0$ and the dispersion parameter $\gamma > 0$ of the negative-binomial model and calculate an estimated, roughly 70%-confidence interval for $\lambda$. What do you observe?

Exercise 4.2 Compound Poisson Distribution

For the total claim amount $S$ of an insurance company we assume $S \sim \text{CompPoi}(\lambda v, G)$, where $\lambda = 0.06$, $v = 10$ and for a random variable $Y$ with distribution function $G$ we have

<table>
<thead>
<tr>
<th>$k$</th>
<th>100</th>
<th>300</th>
<th>500</th>
<th>6'000</th>
<th>100'000</th>
<th>500'000</th>
<th>2'000'000</th>
<th>5'000'000</th>
<th>10'000'000</th>
</tr>
</thead>
</table>

Table 2: Distribution of $Y \sim G$.

Suppose that the insurance company wants to distinguish between

- small claims: claim size $\leq 1'000$,
- medium claims: $1'000 < \text{claim size} \leq 1'000'000$ and
- large claims: claim size $> 1'000'000$.

Let $S_{sc}$, $S_{mc}$ and $S_{lc}$ be the total claims in the small claims layer, in the medium claims layer and in the large claims layer, respectively.

(a) Give definitions of $S_{sc}$, $S_{mc}$ and $S_{lc}$ in terms of mathematical formulas.

(b) Determine the distributions of $S_{sc}$, $S_{mc}$ and $S_{lc}$.

(c) What is the dependence structure between $S_{sc}$, $S_{mc}$ and $S_{lc}$?

(d) Calculate $E[S_{sc}]$, $E[S_{mc}]$ and $E[S_{lc}]$ as well as $\text{Var}(S_{sc})$, $\text{Var}(S_{mc})$ and $\text{Var}(S_{lc})$. Use these values to calculate $E[S]$ and $\text{Var}(S)$.

(e) Calculate the probability that the total claim in the large claims layer exceeds 5 millions.
Exercise 4.3 Method of Moments
We assume that the independent claim sizes $Y_1, \ldots, Y_8$ all follow a Gamma distribution with the same unknown shape parameter $\gamma > 0$ and the same unknown scale parameter $c > 0$ and that we have the following observations for $Y_1, \ldots, Y_8$:

$$x_1 = 7, \quad x_2 = 8, \quad x_3 = 10, \quad x_4 = 9, \quad x_5 = 5, \quad x_6 = 11, \quad x_7 = 6, \quad x_8 = 8.$$ 

Calculate the method of moments estimates of $\gamma$ and $c$. 

Updated: October 4, 2017