ETH Zürich, D-MATH
HS 2017
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Non-Life Insurance: Mathematics and Statistics

Exercise sheet 7

Exercise 7.1 Hill Estimator (R Exercise)
Write an R-Code that samples 300 independent observations from a Pareto distribution with threshold $\theta = 10$ millions and tail index $\alpha = 2$. Use set.seed(100) for reproducibility reasons. Moreover, create a Hill plot and a log-log plot. What do you observe?

Exercise 7.2 Approximations for Compound Distributions
Assume that S has a compound Poisson distribution with expected number of claims $\lambda v = 1\text{'}000$ and claim sizes following a gamma distribution with shape parameter $\gamma = 100$ and scale parameter $c = \frac{1}{10}$. For all $\alpha \in (0,1)$, let q_α denote the α-quantile of S.

(a) Use the normal approximation to estimate $q_{0.95}$ and $q_{0.99}$.

(b) Use the translated gamma approximation to estimate $q_{0.95}$ and $q_{0.99}$.

(c) Use the translated log-normal approximation to estimate $q_{0.95}$ and $q_{0.99}$.

(d) Comment on the results found above.

Exercise 7.3 Akaike Information Criterion and Bayesian Information Criterion
Assume that we have i.i.d. claim sizes $Y_1, \ldots, Y_n$, with $n = 1\text{'}000$, coming from a gamma distribution. Moreover, assume that if we fit a gamma distribution to this data, we obtain the following method of moments estimators and the MLEs:

$$\hat{\gamma}^{\text{MM}} = 0.9794 \quad \text{and} \quad \hat{c}^{\text{MM}} = 9.4249,$$
$$\hat{\gamma}^{\text{MLE}} = 1.0013 \quad \text{and} \quad \hat{c}^{\text{MLE}} = 9.6360.$$

The corresponding log-likelihoods are given by

$$\ell_{\mathbf{Y}}\left(\hat{\gamma}^{\text{MM}}, \hat{c}^{\text{MM}}\right) = 1264.013 \quad \text{and} \quad \ell_{\mathbf{Y}}\left(\hat{\gamma}^{\text{MLE}}, \hat{c}^{\text{MLE}}\right) = 1264.171.$$

(a) Why is $\ell_{\mathbf{Y}}\left(\hat{\gamma}^{\text{MLE}}, \hat{c}^{\text{MLE}}\right) > \ell_{\mathbf{Y}}\left(\hat{\gamma}^{\text{MM}}, \hat{c}^{\text{MM}}\right)$? Which fit should be preferred according to the Akaike Information Criterion (AIC)?

(b) The estimates of γ are very close to 1 and, thus, we could also use an exponential distribution as claim size distribution. For the exponential distribution we obtain the MLE $\hat{c}^{\text{MLE}} = 9.6231$ and the corresponding log-likelihood $\ell_{\mathbf{Y}}\left(\hat{c}^{\text{MLE}}\right) = 1264.169$. According to the AIC and the Bayesian Information Criterion (BIC), should we prefer the gamma model or the exponential model?