Non-Life Insurance: Mathematics and Statistics

Exercise sheet 8

Exercise 8.1 Panjer Algorithm
In this exercise we calculate monthly health insurance premiums for different franchises d using the Panjer algorithm. In particular, we assume that the yearly claim amount

$$S = \sum_{i=1}^{N} Y_i$$

of a given customer is compound Poisson distributed with $N \sim \text{Poi}(1)$ and $Y_1 \stackrel{(d)}{=} k + Z$, where $k = 100$ CHF and $Z \sim \text{LN}(\mu = 7.8, \sigma^2 = 1)$. In health insurance, the policyholder can choose between different franchises $d \in \{300, 500, 1'000, 1'500, 2'000, 2'500\}$. Moreover, he has to pay $\alpha = 10\%$ of the part of the total claim amount that exceeds the franchise d, but only up to a maximum of $M = 700$ CHF. Thus, the yearly amount paid by the customer is given by

$$S_{\text{ins}} = \min\{S, d\} + \min\{\alpha \cdot (S - d)_+, M\}.$$

If we define $\pi_0 = \mathbb{E}[S]$ and $\pi_{\text{ins}} = \mathbb{E}[S_{\text{ins}}]$, then the monthly pure risk premium π is given by

$$\pi = \frac{\pi_0 - \pi_{\text{ins}}}{12}.$$

Calculate π for the different franchises $d = 300, 500, 1'000, 1'500, 2'000$ and $2'500$ using the Panjer algorithm. In order to do that, discretize the translated log-normal distribution using a span of $s = 10$ and putting all the probability mass to the upper end of the intervals.

Exercise 8.2 Variance Loading Principle
We would like to insure the following car fleet:

i	v_i	λ_i	$\mathbb{E}[Y_1^{(i)}]$	$\text{Vco}(Y_1^{(i)})$
passenger car	40	25%	2'000	2.5
delivery van	30	23%	1'700	2.0
truck	10	19%	4'000	3.0

Assume that the total claim amounts for passenger cars, delivery vans and trucks can be modeled by independent compound Poisson distributions.

(a) Calculate the expected claim amount of the car fleet.

(b) Calculate the premium for the car fleet using the variance loading principle with $\alpha = 3 \cdot 10^{-6}$.

Exercise 8.3 Panjer Distribution
Let N be a random variable that has a Panjer distribution with parameters $a, b \in \mathbb{R}$. Calculate $\mathbb{E}[N]$ and $\text{Var}(N)$. What can you say about the ratio of $\text{Var}(N)$ to $\mathbb{E}[N]$?