# Non-Life Insurance: Mathematics and Statistics

## Exercise sheet 9

**Exercise 9.1 Utility Indifference Price**
In this exercise we calculate the premium for the accident insurance of a given company COMP using the utility indifference price principle. We suppose that all employees of COMP have been divided into two groups, depending on their work, and that the total claim amounts $S_1$ and $S_2$ of the two groups are independent and compound Poisson distributed:

| Group $i$ | $v_i$ | $\lambda_i$ | $Y_1^{(i)}$ |
|---|---|---|---|
| Group 1 | 2'000 | 50% | $\Gamma(\gamma = 20, c = 0.01)$ |
| Group 2 | 10'000 | 10% | expo(0.005) |

We write $S = S_1 + S_2$ for the total claim amounts of COMP. Let $c_0$ be the initial capital of the insurance company that sells accident insurance to COMP.

(a) Let $u$ be a utility function. Show that if the utility indifference price $\pi = \pi(u, S, c_0)$ exists, then it is unique and satisfies $\pi > \mathbb{E}[S]$.

(b) Calculate $\mathbb{E}[S]$.

(c) Calculate $\pi$ using the utility indifference price principle for the exponential utility function with parameter $\alpha = 1.5 \cdot 10^{-6}$.

(d) What happens to $\pi$ if we replace the compound Poisson distributions by Gaussian distributions with the same corresponding first two moments?

**Exercise 9.2 Value-at-Risk and Expected Shortfall**
Suppose that for the yearly claim amount $S$ of an insurance company in a given line of business we have $S \sim \mathrm{LN}(\mu, \sigma^2)$ with $\mu = 20$ and $\sigma^2 = 0.015$. Moreover, we set the cost-of-capital rate $r_{\mathrm{CoC}} = 6\%$. Then the premium $\pi_{\mathrm{CoC}}$ for the considered line of business using the cost-of-capital pricing principle is given by

$$\pi_{\mathrm{CoC}} = \mathbb{E}[S] + r_{\mathrm{CoC}}\, \rho(S - \mathbb{E}[S]),$$

where $\rho$ is a risk measure.

(a) Calculate $\pi_{\mathrm{CoC}}$ using the VaR risk measure at security level $1 - q = 99.5\%$.

(b) Calculate $\pi_{\mathrm{CoC}}$ using the expected shortfall risk measure at security level $1 - q = 99\%$.

(c) Which security level is needed such that $\pi_{\mathrm{CoC}}$ using the VaR risk measure is equal to the price calculated in (b)?

(d) Let $U$ and $V$ be two independent copies of $\log S$. Show that on the one hand

$$\mathrm{VaR}_{0.45}(U + V) > \mathrm{VaR}_{0.45}(U) + \mathrm{VaR}_{0.45}(V),$$

but on the other hand

$$\mathrm{VaR}_{0.55}(U + V) < \mathrm{VaR}_{0.55}(U) + \mathrm{VaR}_{0.55}(V).$$

In particular, the VaR is not subadditive, and hence not coherent.

**Exercise 9.3 Esscher Premium**
Let $S$ be a random variable with distribution function $F$ and moment generating function $M_S$. Assume that there exists $r_0 > 0$ such that $M_S(r) < \infty$ for all $r \in (-r_0, r_0)$. For $\alpha \in (0, r_0)$, let $\pi_\alpha$ denote the Esscher premium of $S$.

(a) Show that if $S$ is non-deterministic, then $\pi_\alpha$ is strictly increasing in $\alpha$.

(b) Show that the Esscher premium for small values of $\alpha$ boils down to a variance premium principle.

(c) Suppose that $S \sim \text{CompPoi}(\lambda v, G)$, where $\lambda v > 0$ and $G$ is the distribution function of a gamma distribution with shape parameter $\gamma > 0$ and scale parameter $c > 0$. For which values of $\alpha$ does $\pi_\alpha$ exist? Calculate $\pi_\alpha$ where it is defined.