Exercices -

Sheet 1

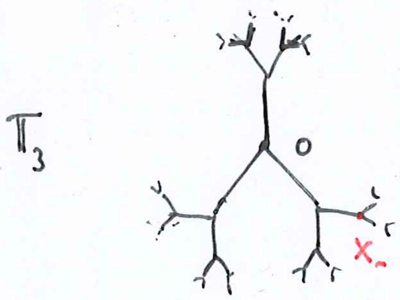
Exercice 1

Prove that the isoperimetric constant on $\mathbb{Z}^d$ is

$$\phi = 0 .$$

Exercice 2

. Consider the simple random walk (X_n) on $\mathbb{T}_d$



Recall that . $|X_n|$ is the graph distance between o and X_n.

. $p_n = \mathbb{P}[X_n = o]$

(i) Show that for every $n \geq 0$

$$E[|X_{n+1}|] = p_n + (1-p_n) \cdot \frac{(d-2)}{d} + E[|X_n|]$$

(ii) Deduce that $\dfrac{E[|X_n|]}{n} \xrightarrow[n \to \infty]{} \dfrac{d-2}{d}$.

Exercice 3

Consider the simple random walk (X_n) on $\mathbb{Z}^d$.

Prove that there exists a constant $c > 0$ such that

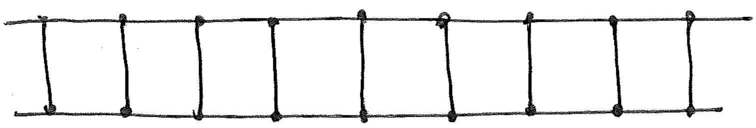
$$\frac{E[|X_n|]}{\sqrt{n}} \xrightarrow[n \to \infty]{} c .$$

Exercice 4

Consider the ladder graph with vertex and edge sets

$$V = \mathbb{Z} \times \{0,1\}$$

$$E = \{\{(i,\varepsilon),(i+1,\varepsilon)\},\ i \in \mathbb{Z},\ \varepsilon \in \{0,1\}\} \cup \{\{(i,0),(i,1)\},\ i \in \mathbb{Z}\}$$



Prove that for Bernoulli percolation on this graph

$$p_c = 1 .$$