

Exercises :  
Sheet 10 .

Ex 1: Let  $G$  be a transitive graph. Prove that

$$(LL(G) \text{ has LP}) \Leftrightarrow (G \text{ is recurrent})$$

Ex 2: Percolation : basic questions.

- Why are  $x \leftrightarrow y$  and  $x \leftrightarrow \infty$  measurable?
- Why is  $w \mapsto N(w)$  measurable?
- Let  $w \in \{0,1\}^E$ . Check that

$$(|C_0(w)| = \infty) \Leftrightarrow \left( \begin{array}{l} \exists (\gamma_n)_{n \geq 0} \text{ infinite path } (\gamma_i \neq \gamma_j \text{ if } i \neq j) \\ \text{o.t. } \forall n \quad w(\gamma_n \gamma_{n+1}) = 1 \end{array} \right)$$

Ex 3: Let  $p \in (0,1)$ . What is the largest constant  $c > 0$  s.t

$$\forall A \in \mathcal{F} \quad P_p(\pi^* A) \geq c P_p(A) \quad ?$$