

- EXERCICES -

SHEET 3

(*) [N] Exercice 1

Let G be an infinite, locally-finite, transitive graph.
Show that:

$(G \text{ non amenable}) \implies (G \text{ has exponential growth}).$

Exercice 2

We consider the lamplighter graph $G = LL(\mathbb{Z})$.
Let T_n be the set of vertices of G at distance exactly n from 0 and obtained from 0 by using only the moves "move right", "turn the lamp on".
Show that $\forall n$

$$|T_{n+2}| = |T_{n+1}| + |T_n|$$

Deduce that the volume growth exponent of G is

$$v = \log\left(\frac{1+\sqrt{5}}{2}\right).$$

Exercice 3

Prove that $LL(G)$ is transitive whenever G is transitive.

Exercice 4

Let G be a graph and assume that it contains an odd closed walk (i.e. $\exists (x_0, \dots, x_n)$ s.t. $x_i \sim x_{i+1}$, $x_0 = x_n$, n odd).
Show that G contains an odd cycle.