

- EXERCICES -

SHEET 4

G on finite locally-finite connected transitive graph, $o \in V$.

Exercice 1

1. Prove that for every finite set $A \subset V$,

$$P_{2m}(o) \geq \frac{1}{|A|} P_o[X_{2m} \in A]$$

2. Prove that for the SRW on $G = LL(\mathbb{Z})$, we have

$$\forall m \quad P_{2m}(o) \geq \frac{c}{2^{4m}}.$$

Deduce that $\rho(G) = 0$.

Exercice 2

Let $f: V \rightarrow \mathbb{R}$ bounded. Prove that $\forall m \geq 0 \quad \forall x \in V$,

$$E_x[f(X_{2m})] = \frac{1}{d} \sum_{y \sim x} E_y[f(X_m)]$$

Exercice 3 [bounds on the spectral radius] $d = \deg(G)$

1. [Level 1] Prove that $\frac{1}{d} \leq \rho(G) \leq 1$

2. [Level 2] Prove that

$$\rho(G) \geq \frac{2\sqrt{d-2}}{d}$$

Exercice 4

Prove the Rayleigh quotient. for $\|P\|_2$.