

EXERCICES :

Sheet 5

Exercise 1 :

Define the operator $\text{div} : \mathcal{D}_K \rightarrow \mathcal{E}_K$ by

$$\forall \theta \in \mathcal{D}_K, \forall (x, y) \in \tilde{E} \quad \boxed{(\text{div}(\theta))(x) = \sum_{b \sim x} \theta((x, y))}$$

1.) Prove that div and ∇ are adjoint of each other,

$$\text{ie } \forall f \in \mathcal{E}_K \quad \forall \theta \in \mathcal{D}_K \quad \langle \theta, \nabla f \rangle_{\tilde{E}} = \langle \text{div} \theta, f \rangle.$$

2.) Prove that $\text{Id} - P = \frac{1}{d} \cdot (\text{div} \circ \text{grad})$ and deduce that $\forall f, g \in \mathcal{E}_K$

$$\langle (\text{Id} - P)f, g \rangle = \frac{1}{d} \langle \nabla f, \nabla g \rangle.$$