

- EXERCICES -

Sheet 7.

Exercise 1

Prove that for the lamplighter graph  $G = LL(\mathbb{Z})$ , there exists  $c_1, c_2 > 0$  such that

$$\forall u > 1 \quad \frac{c_1}{\log(u)} \leq \varphi(u) \leq \frac{c_2}{\log u}.$$

(\*) Exercise 2

1) Prove that  $(p_{2k}(0,0))_{k \geq 0}$  is decreasing in  $k$ .

2) Prove that for every  $n \geq 0$   $p_{2n}(0,0) \leq 2 q_{2n}(0,0)$ .

Exercise 3

Prove that  $\sum_{n=0}^{\infty} q_n(0,0) = 2 \sum_{n=0}^{\infty} p_n(0,0)$ .

Exercise 4

Consider the evolving sets  $(S_n)$  associated to the lazy RW on  $\mathbb{Z}^2$  (starting from  $S_0 = \{0\}$ ). Compute

$$|\{S \subset V : \mathbb{P}[S_n = S] > 0\}|.$$

Exercise 5

Prove that  $q_n(0,0) \xrightarrow{n \rightarrow \infty} 0$ . Deduce that  $\mathbb{P}[S_n = \emptyset] \xrightarrow{n \rightarrow \infty} 1$ .