

- EXERCICES -

- SHEET 8 -

Exercise 1:

1. Complete the proof of the main theorem in Section 5, Chap 3 based on the steps presented in class.

2. [Random walk on \mathbb{Z} , killed at 0]

Let $(Z_i)_{i \in \mathbb{N}}$ iid s.t. $P[Z_i = -1] = P[Z_i = +1] = \frac{1}{2}$

$$X_n = 1 + \sum_{i=1}^n Z_i \quad (X_0 = 1)$$

Define $M_n = X_n \wedge T$ where $T = \inf \{n : X_n = 0\}$

Check that M_n is a martingale, and prove using the same approach as in the course that

$$P[M_n \neq 0] \leq \frac{c}{\sqrt{n}} \quad (c > 0 \text{ constant})$$

Exercise 2

Prove the corollary in Section 5, Chap. 3.