Mathematical Finance

Exercise sheet 5

Let $\mathcal{S}$ denote the set of semimartingales and $\mathbb{S}_1 := \{H \in \mathbb{S} : ||H||_\infty \leqslant 1\}$ the unit ball of simple predictable processes. The Emery topology is a topology on $\mathcal{S}$ generated by the metric

$$d_E(X, Y) := \sum_{n=1}^{\infty} 2^{-n} \sup_{H \in \mathbb{S}_1} E\left[1 \wedge \sup_{t \leqslant n} |(H \bullet (X - Y))_t|\right].$$

Exercise 5.1 Show that

(a) $\mathcal{S}$ endowed with the Emery topology is a topological vector space.

(b) $\mathcal{S}$ is closed in the Emery topology and complete with respect to the metric d_E.

Exercise 5.2 Show that the Emery topology is invariant under an equivalent change of measure.

Exercise 5.3 Let the set of adapted càdlàg process $\mathbb{L}$ be endowed with the u.c.p. topology and the set of semimartingales $\mathcal{S}$ be endowed with the Emery topology, and let X be a semimartingale. Show that

$$J_X : \mathbb{L} \ni Y \mapsto (Y \bullet X) \in \mathcal{S}$$

is continuous.

Exercise 5.4 (Python) Let B and C be two independent Brownian motions. Compute and plot the distribution of

$$A_t := \frac{1}{2} \int_0^t \int_0^u (dB_s dC_u - dC_s dB_u)$$

for some $t > 0$.