ETH Zürich, HS 2017
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Mathematical Finance

Exercise sheet 7

Exercise 7.1 Show that a sublinear functional on a vector space X is linear if and only if it dominates exactly one linear functional on X.

Exercise 7.2 Show that in finite discrete time we have

$$(NA) \implies (NUPBR).$$

Exercise 7.3 Give an example of a family of random variables that is bounded in probability but whose convex hull is not bounded in probability.

Exercise 7.4 Construct a semimartingale S with a Doob-Meyer decomposition $S = M + [M, M]$ such that the local martingale $\mathcal{E}(-M)$ is strict, but there exists an equivalent martingale measure for S.

Exercise 7.5 (Python) Let B be a standard Brownian motion motion and consider a market consisting of three assets $S^0 \equiv 1$, $S^1_t = \exp(B_t)$ and $S^2_t = \exp\left(\frac{1}{2}B_t\right)$, $t \in [0, T]$, for some $0 < T < \infty$. Verify numerically that the market admits scalable arbitrage.