

Mathematical Finance

Exercise sheet 8

Exercise 8.1 Show that

- (a) A local martingale is a sigma-martingale.
- (b) A sigma-martingale which is also a special semimartingale is a local martingale.

Exercise 8.2 Give an example of a sigma-martingale that is not a local martingale.

Exercise 8.3 Give an example of a large financial market that does not admit an equivalent sigma-martingale measure but for which there exists an equivalent separating measure.

Exercise 8.4 (Python) Assume Black-Scholes dynamics for S , say $(r, \mu, \sigma) = (0, 0, 1)$, and find the hedging strategy H for the log-contract g whose discounted payoff is given by

$$g(S_T) = \log \frac{S_T}{S_0} + \frac{1}{2} \sigma^2 T.$$

Compare numerically the value of $g(S_T)$ to $(H \bullet S)_T$ at $T = 1$.