Mathematical Finance

Exercise sheet 11

Exercise 11.1 Give an example of a bounded predictable process H and a martingale X such that $(H \bullet X)$ is not a martingale.

Exercise 11.2 Let the financial market $S = (S_t)_{t=0}^T$ be defined over the *finite* filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t=0}^T, P)$ and satisfy $\mathcal{M}^e(S) = \{Q\}$, and let U be a utility function satisfying the Inada conditions. Consider the value functions

$$u(x) = \sup_{X_T \in C(x)} E[U(X_T)] \text{ and } v(y) = E\left[V\left(y\frac{dQ}{dP}\right)\right],$$

where V is the convex conjugate of U and $C(x) = \{X_T \in L^0(\Omega, \mathcal{F}_T, P) : E_Q[X_T] \leqslant x\}$. Show that the optimizers $\widehat{X}_T(x)$ and $\widehat{y}(x)$ satisfy $U'\left(\widehat{X}_T(x)\right) = \widehat{y}(x)\frac{dQ}{dP}$ for each $x \in \mathrm{dom}(U)$.

Exercise 11.3 Assume a Black-Scholes model and compute the discounted value process V for a power option whose undiscounted payoff at time T is $h(\widetilde{S}_T) = \widetilde{S}_T^p$ for some $p \in \mathbb{R}$.

Exercise 11.4 (Python) Compute the hedging strategy for the power option and examine the hedging error numerically.