### 5.1. Lipschitz vs. bounded weak derivative

The domain $\Omega$ must be non-convex.

### 5.2. A tent for Rudolf L.

Who is Rudolf L.?

### 5.3. Capacity and Hausdorff measure

**(a)** Work with the definitions of Hausdorff measure and capacity. You may use that for any $r > 0$ there exists some $\psi \in C_c^\infty(B_{3r})$ satisfying $\psi = 1$ in $B_{2r}$ and $|\nabla\psi| \leq \frac{2}{r}$. The statement of Problem 5.6 (b) may come in handy.

**(b)** Recall Satz 8.1.1.

### 5.4. Traceless

Find a sequence of functions $u_k \in C^0(\overline{\Omega})$ satisfying $u_k|_{\partial\Omega} \equiv 1$ and $\|u_k\|_{L^p(\Omega)} \xrightarrow{k\to\infty} 0$.

### 5.5. Traces of weak derivatives

**(a)** For $1 \leq p < \infty$ apply Fubini's theorem and for $p = \infty$, argue with Lipschitz continuity (Korollar 8.3.1). Then use the same trick as in Problem 4.5 (a).

**(b)** Apply part (a) and use Lemma 7.3.1.

### 5.6. Positive and negative part

**(a)** For $\varepsilon > 0$ consider the function $G_\varepsilon \circ u$, where $G_\varepsilon \in C^1(\mathbb{R})$ is given by

$$G_\varepsilon(y) = \begin{cases} \sqrt{y^2+\varepsilon^2} - \varepsilon & \text{for } y \geq 0, \\ 0 & \text{for } y < 0. \end{cases}$$

**(b)** Use part (a).

**(c)** Notice that $u = u_+ - u_-$ and use part (a).

**(d)** Use part (c) but be careful: unless $\Omega$ is bounded, constants are only in $W^{1,p}_{\mathrm{loc}}(\Omega)$.