SHEET 1.

Exercise 1

Prove that $(A \leftrightarrow B)$ is measurable for $A, B \subset \mathbb{Z}^d$.

Prove that

$$F : \{0,1\}^E \longrightarrow \mathbb{N} \cup \{+\infty\} \quad \text{is measurable.}$$
$$\omega \longmapsto |C_x(\omega)|$$

Exercise 2

An <u>infinite open path</u> from 0 (in ω) is a sequence $(\gamma_i)_{i \in \mathbb{N}}$ of distinct vertices such that

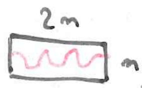
$$\gamma_0 = 0 \quad , \quad \forall i \geq 1 \ \gamma_{i-1} \sim \gamma_i \ . \ \text{and} \ \omega(\gamma_{i-1}, \gamma_i) = 1 .$$

Prove that the event

$$A = \{ \text{there exists an infinite open path from } 0 \} .$$

is measurable.

Exercise 3.

Let $q_n = 1 - \mathbb{P}_p \Big[$  $\Big]$.

Prove that one of the following holds.

1.) $\exists c > 0$ s.t $\forall n$ $q_n \geq c$

2) $\exists c > 0$ s.t $\forall n$ $q_n \leq e^{-cn}$.