SHEET 10

Exercise 1. In this exercise we consider percolation on $\mathbb{Z}^2$.

Let $p > p_c$. Using duality, prove that

$$\xi(p) = \left(\lim_{n \to \infty} - \frac{\log\left(\mathbb{P}_p[0 \leftrightarrow \partial\Lambda_n,\ 0 \leftrightarrow \infty]\right)}{n} \right)^{-1}$$

is well defined and show that $\xi(p) = \frac{1}{2}\,\xi(1-p)$.

"subcritical correlation length".

Exercise 2.

Consider percolation on $\mathbb{Z}^d$, $d \geq 2$.

Let $p > p_c(\mathbb{Z}^d)$. Prove that there exist $c_1, c_2 > 0$ s.t.

$$\forall x \in \mathbb{Z}^d \qquad e^{-c_0\|x\|} \leq \mathbb{P}_p[0 \leftrightarrow x,\ 0 \not\leftrightarrow \infty] \leq e^{-c_1\|x\|}$$