

SHEET 2

* Exercise 1

Consider percolation on $(\mathbb{Z}^d, \mathcal{E})$.

Let A, B be two increasing events. Prove that

$$P_p(A \cap B) \geq P_p(A) \cdot P_p(B)$$

Let A be an increasing event, B a decreasing event. Prove that

$$P_p(A \cap B) \leq P_p(A) P_p(B).$$

* Exercise 2

Let $k \geq 2$, $\varepsilon > 0$. Let A_1, \dots, A_k be k increasing events.

Assume that

$$P_p\left(\bigcup_{1 \leq i \leq k} A_i\right) \geq 1 - \varepsilon$$

Prove that

$$\max_{1 \leq i \leq k} P_p(A_i) \geq 1 - \varepsilon^{1/k}.$$

Exercise 3

Let $x, y \in \mathbb{Z}^d$ s.t. $x \neq y$. Prove that $f(p) := P_p(x \leftrightarrow y)$ is strictly increasing in p .