. SHEET 3 .

Exercise 1

$G=(V,E)$ finite graph. Let $A \subset \{0,1\}^E$ be an increasing event.

1) Let $\omega \in A$. Prove that there exists a witnes I for A in ω such that $\forall e \in E \;\; \omega(e)=1$.

2) Let A, B be two increasing events. Prove that

$$A \circ B = \left\{ \omega : \exists\, I, J \text{ disjoint } \underline{\text{open}} \text{ such that } \begin{array}{l} I \text{ wit. } A \text{ in } \omega \\ J \text{ wit. } B \text{ in } \omega \end{array} \right\}$$

Exercise 2

Let A be an increasing event, B a decreasing event.

Prove that $A \circ B = A \cap B$.

Deduce that $\mathbb{P}_p[A \circ B] \leq \mathbb{P}_p[A]\,\mathbb{P}_p[B]$ in this case.

Exercise 3

Consider percolation on $(\mathbb{Z}^d, E)$. Let $\theta(p) = \mathbb{P}_p[0 \leftrightarrow \infty]$.

Define the random variable $X_n = \frac{1}{|\Lambda_n|} \sum_{x \in \Lambda_n} \mathbb{1}_{x \leftrightarrow \infty}$.

Prove that

$$\lim_{n \to \infty} X_n = \theta(p) \quad \text{in probability.}$$

(Hint: look at the expectation and the variance of X_n)