

Exercise 1

Define for  $p \in [0, p_c]$   $\tilde{\mathcal{F}}(p) = \min \{n \geq 1 : \phi_p(\Lambda_n) \leq \frac{1}{e}\}$ .

Prove that  $\exists C = C(d)$  s.t.

$$\forall p \in [0, p_c] \quad \mathcal{F}(p) \leq \tilde{\mathcal{F}}(p) \leq 1 + C \mathcal{F}(p) \log(2 + \mathcal{F}(p))$$

Exercise 2

Prove that  $\mathcal{F}: [0, p_c] \longrightarrow [0, \infty]$  is strictly increasing.  
 $p \longmapsto \mathcal{F}(p)$